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A detailed analytical model of random polycrystals of porous laminates has been developed. This approach permits detailed calculations of poromechanics constants as well as transport coefficients. The resulting earth reservoir model allows studies of both geomechanics and fluid permeability to proceed semi-analytically. Rigorous bounds of the Hashin-Shtrikman type provide estimates of overall bulk and shear moduli, and thereby also provide rigorous error estimates for geomechanical constants obtained from up-scaling based on a self-consistent effective medium method. The influence of hidden or unknown microstructure on the final results can then be evaluated quantitatively. Descriptions of the use of the model and some examples of typical results on the poromechanics of such a heterogeneous reservoir are presented.

1 INTRODUCTION

An explosion of new results on rigorous bounding methods (Milton 2002; Torquato 2002) has been seen over the last fifty years. Effective medium theory, although very useful in many circumstances, nevertheless has not seen a similar explosion of new results. So a question that naturally arises is whether we can construct new effective medium formulas directly from these known bounds? Skeptics will immediately ask: Why do we need to do this at all if the bounds are available? But the answer to this question is most apparent in poromechanics, where the bounds are often too far apart to be of much practical use.

Hill (1952) was actually the first to try constructing estimates from bounds. He showed that the Voigt (1928) and Reuss (1929) averages/estimates in elasticity were in fact upper and lower bounds, respectively. Then he proceeded to suggest that estimates of reasonable accuracy were given by the arithmetic or geometric means obtained by averaging these two bounds together. Thus, the Voigt-Reuss-Hill estimates were born. Better bounds than the Voigt and Reuss bounds are now known and no doubt some attempts to update Hill's approach have been made. However, to make a direct connection to traditional approaches of effective medium theory, we apply a more technical approach here in order to obtain estimates of up-scaled constants using the known analytical structure of the bounds, especially for Hashin and Shtrikman (1962) bounds. When this mathematical structure is not known, then it proves very worthwhile to expend the effort required to find this structure. Indeed many of these bounds, which are complex nonlinear functions of the parameters, have been expressed only algorithmically — not as analytical formulas. But, in some cases it has been possible to carry the analysis further than was done in the published literature. When this is true, a self-consistent effective medium formula is then straightforward to obtain from the resulting expressions. Then, as expected, resulting predictions are consistent with the bounds.

In Section 2, results from a double-porosity geomechanics analysis are presented. These results are general and do not depend explicitly on the spatial arrangement of the two porous constituents. Microstructure enters these formulas only through the overall drained bulk modulus K^*. Then, in Section 3, a preferred microstructure — that of a locally layered medium — is imposed. This microstructure has the advantage that it forms hexagonal (or transversely isotropic) "crystals" locally. Then, if we assume these crystals are jumbled together randomly to form an overall isotropic medium, we have the "random polycrystal of porous laminates" reservoir model. Hashin-

Shtrikman bounds are known for such polycrystals composed of grains having hexagonal symmetry. So bounds are easily found. From the form of the bounds, we also obtain estimates of both overall bulk modulus and shear modulus, thus completing the semi-analytical poromechanics model. Examples are computed in Section 4, and results summarized in the final section.

2 DOUBLE-POROSITY GEOMECHANICS

The main results can be derived using uniform expansion, or self-similar, methods analogous to ideas used in thermoelasticity by Cribb (1968) and in single-porosity poroelasticity by Berryman and Milton (1991). Cribb's method provided a simpler derivation of earlier results on thermoelastic expansion coefficients. Our results also provide a simpler derivation of results obtained by Berryman and Pride (2002) for the double-porosity coefficients. Related methods in micromechanics are called "the theory of uniform fields" by some authors (Dvorak and Benveniste 1997).

First assume two distinct phases at the macroscopic level: a porous matrix phase with the effective properties $K^{(1)}$, $G^{(1)}$, $K_m^{(1)}$, $\phi^{(1)}$ (which are drained bulk and shear moduli, grain/mineral bulk modulus, and porosity of phase 1 with analogous definitions for phase 2), occupying volume fraction $V^{(1)}/V = v^{(1)}$ of the total volume and a macroscopic crack or joint phase occupying the remaining fraction of the volume $V^{(2)}/V = v^{(2)} = 1 - v^{(1)}$.

There are three distinct pressures: confining pressure δp_c, pore-fluid pressure $\delta p_f^{(1)}$, and joint-fluid pressure $\delta p_f^{(2)}$. Treating $\delta p_c, \delta p_f^{(1)}$, and $\delta p_f^{(2)}$ as the independent variables in our double porosity theory, we define the dependent variables $\delta e \equiv \delta V/V$, $\delta\zeta^{(1)} = (\delta V_\phi^{(1)} - \delta V_f^{(1)})/V$, and $\delta\zeta^{(2)} = (\delta V_\phi^{(2)} - \delta V_f^{(2)})/V$, which are respectively the total volume dilatation, the increment of fluid content in the matrix phase, and the increment of fluid content in the joints. The fluid in the matrix is the same as that in the cracks or joints, but the two fluid regions may be in different states of average stress and, therefore, need to be distinguished by their respective superscripts.

Linear relations among strain, fluid content, and pressure take the symmetric form

$$\begin{pmatrix} \delta e \\ -\delta\zeta^{(1)} \\ -\delta\zeta^{(2)} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} \begin{pmatrix} -\delta p_c \\ -\delta p_f^{(1)} \\ -\delta p_f^{(2)} \end{pmatrix}, \quad (1)$$

following Berryman and Wang (1995) amd Lewallen and Wang (1998). It is easy to check that $a_{11} = 1/K^*$, where K^* is the overall drained bulk modulus of the system. We now find the remaining five constants for a binary composite system.

The components of the system are themselves porous materials 1 and 2, but each is assumed to be what we call a "Gassmann material" satisfying

$$\begin{pmatrix} \delta e^{(1)} \\ -\delta\zeta^{(1)}/v^{(1)} \end{pmatrix} =$$

$$\frac{1}{K^{(1)}} \begin{pmatrix} 1 & -\alpha^{(1)} \\ -\alpha^{(1)} & \alpha^{(1)}/B^{(1)} \end{pmatrix} \begin{pmatrix} -\delta p_c^{(1)} \\ -\delta p_f^{(1)} \end{pmatrix} \quad (2)$$

for material 1 and a similar expression for material 2. The new constants appearing on the right are the drained bulk modulus $K^{(1)}$ of material 1, the corresponding Biot and Willis (1957) parameter $\alpha^{(1)}$, and the Skempton (1954) coefficient $B^{(1)}$. The volume fraction $v^{(1)}$ appears here to correct for the difference between a global fluid content and the corresponding local variable for material 1. The main special characteristic of a Gassmann (1951) porous material is that it is composed of only one type of solid constituent, so it is "microhomogeneous" in its solid component, and in addition the porosity is randomly, but fairly uniformly, distributed so there is a well-defined constant porosity $\phi^{(1)}$ associated with material 1, etc.

To proceed, we ask this question: Is it possible to find combinations of $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}$, $\delta p_f^{(1)}$, and $\delta p_f^{(2)}$ so that the expansion or contraction of the system is spatially uniform or self-similar? Or, can we find uniform confining pressure δp_c, and pore-fluid pressures $\delta p_f^{(1)}$ and $\delta p_f^{(2)}$, so all these scalar conditions can be met simultaneously? If so, then results for system constants can be obtained purely algebraically without ever having to solve equilibrium equations. We initially set $\delta p_c = \delta p_c^{(1)} = \delta p_c^{(2)}$, as this condition of uniform confining pressure is clearly necessary for this self-similar thought experiment to be a valid solution of stress equilibrium equations.

So, the first condition to be considered is the equality of the strains of the two constituents:

$$\delta e^{(1)} = -\frac{1}{K^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)}) =$$

$$\delta e^{(2)} = -\frac{1}{K^{(2)}}(\delta p_c - \alpha^{(2)}\delta p_f^{(2)}). \quad (3)$$

If this condition is satisfied, then the two constituents are expanding or contracting at the same rate and it is clear that self-similarity prevails. If we imagine that δp_c and $\delta p_f^{(1)}$ are fixed, then we need an appropriate value of $\delta p_f^{(2)}$, so that (3) is satisfied. This requires that

$$\delta p_f^{(2)} = \delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)}) =$$

$$\frac{1 - K^{(2)}/K^{(1)}}{\alpha^{(2)}}\delta p_c + \frac{\alpha^{(1)}K^{(2)}}{\alpha^{(2)}K^{(1)}}\delta p_f^{(1)}, \quad (4)$$

showing that, for undrained conditions, $\delta p_f^{(2)}$ can almost always be chosen so the uniform expansion takes place.

Using (4), we now eliminate $\delta p_f^{(2)}$ from the remaining equality so

$$\delta e = -\left[a_{11}\delta p_c + a_{12}\delta p_f^{(1)} + a_{13}\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})\right]$$

$$= \delta e^{(1)} = -\frac{1}{K^{(1)}}(\delta p_c - \alpha^{(1)}\delta p_f^{(1)}), \qquad (5)$$

where $\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$ is given by (4). Making the substitution and then noting that δp_c and $\delta p_f^{(1)}$ were chosen independently and arbitrarily, we find the resulting coefficients must each vanish. The equations we obtain are

$$a_{11} + a_{13}\left(1 - K^{(2)}/K^{(1)}\right)/\alpha^{(2)} = 1/K^{(1)} \qquad (6)$$

and

$$a_{12} + a_{13}\left(\alpha^{(1)}K^{(2)}/\alpha^{(2)}K^{(1)}\right) = -\alpha^{(1)}/K^{(1)}. \quad (7)$$

With a_{11} known, (6) can be solved directly, giving

$$a_{13} = -\frac{\alpha^{(2)}}{K^{(2)}}\left(\frac{1 - K^{(1)}/K^*}{1 - K^{(1)}/K^{(2)}}\right). \qquad (8)$$

Similarly, with a_{13} known, substituting into (7) gives

$$a_{12} = -\frac{\alpha^{(1)}}{K^{(1)}}\left(\frac{1 - K^{(2)}/K^*}{1 - K^{(2)}/K^{(1)}}\right). \qquad (9)$$

So, three of the six coefficients are known.

To evaluate the remaining coefficients, we consider what happens to fluid increments during self-similar expansion. We treat only material 1, but the equations for material 2 are completely analogous. From the preceding equations,

$$\delta\zeta^{(1)} = a_{12}\delta p_c + a_{22}\delta p_f^{(1)} + a_{23}\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$$

$$= \frac{v^{(1)}}{K^{(1)}}\left[-\alpha^{(1)}\delta p_c + (\alpha^{(1)}/B^{(1)})\delta p_f^{(1)}\right]. \qquad (10)$$

Again substituting for $\delta p_f^{(2)}(\delta p_c, \delta p_f^{(1)})$ from (4) and noting that the resulting equation contains arbitrary values of δp_c and $\delta p_f^{(1)}$, the coefficients of these terms must vanish separately. Resulting equations are $a_{12} + a_{23}(1 - K^{(2)}/K^{(1)})/\alpha^{(2)} = -\alpha^{(1)}v^{(1)}/K^{(1)}$, and $a_{22} + a_{23}\left(\alpha^{(1)}K^{(2)}/\alpha^{(2)}K^{(1)}\right) = \alpha^{(1)}v^{(1)}/B^{(1)}K^{(1)}$. Solving these, we obtain

$$a_{23} = \frac{K^{(1)}K^{(2)}\alpha^{(1)}\alpha^{(2)}}{(K^{(2)} - K^{(1)})^2}\left[\frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}\right], \qquad (11)$$

and

$$a_{22} = \frac{v^{(1)}\alpha^{(1)}}{B^{(1)}K^{(1)}}$$

$$-\left(\frac{\alpha^{(1)}}{1 - K^{(1)}/K^{(2)}}\right)^2\left[\frac{v^{(1)}}{K^{(1)}} + \frac{v^{(2)}}{K^{(2)}} - \frac{1}{K^*}\right]. \qquad (12)$$

Performing the corresponding calculation for $\delta\zeta^{(2)}$ produces formulas for a_{32} and a_{33}. Since (11) is already symmetric in component indices, the formula for a_{32} provides nothing new. The formula for a_{33} is easily seen to be identical in form to a_{22}, but indices 1 and 2 are interchanged.

All five of the nontrivial coefficients of double porosity are now determined.

These results show how the constituent properties K, α, B up-scale at the macrolevel for a two-constituent composite. We find

$$\alpha = -\frac{a_{12} + a_{13}}{a_{11}}$$

$$= \frac{\alpha^{(1)}(K^* - K^{(2)}) + \alpha^{(2)}(K^{(1)} - K^*)}{K^{(1)} - K^{(2)}}, \qquad (13)$$

and

$$\frac{1}{B} = -\frac{a_{22} + 2a_{23} + a_{33}}{a_{12} + a_{13}}. \qquad (14)$$

Note that all important formulas [(8),(9),(11)-(14)] depend on the overall bulk modulus K^* of the system. This quantity must be determined independently either by experiment or by another analytical method.

3 UP-SCALING RANDOM POLYCRYSTALS OF POROUS LAMINATES

3.1 *Elasticity of layered materials*

Next, to determine the overall bulk and shear moduli of the reservoir, assume a typical building block of the random system is a small grain of laminate material whose elastic response for a transversely isotropic (hexagonal) system can be described locally by:

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{33}\\ \sigma_{23}\\ \sigma_{31}\\ \sigma_{12}\end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & & & \\ c_{12} & c_{11} & c_{13} & & & \\ c_{13} & c_{13} & c_{33} & & & \\ & & & 2c_{44} & & \\ & & & & 2c_{44} & \\ & & & & & 2c_{66}\end{pmatrix}\begin{pmatrix}e_{11}\\ e_{22}\\ e_{33}\\ e_{23}\\ e_{31}\\ e_{12}\end{pmatrix}, \qquad (15)$$

where σ_{ij} are the usual stress components for $i, j = 1 - 3$ in Cartesian coordinates, with 3 (or z) being the axis of symmetry (the lamination direction for such a layered material). Displacement u_i is then related to strain component e_{ij} by $e_{ij} = (\partial u_i/\partial x_j +$

$\partial u_j/\partial x_i)/2$. This definition introduces some convenient factors of two into the $44, 55, 66$ components of the stiffness matrix shown in (15).

For definiteness we also assume that the stiffness matrix in (15) arises from the lamination of N isotropic constituents having bulk and shear moduli K_n, μ_n, in the $N > 1$ layers present in each building block. It is important that the thicknesses d_n always be in the same proportion in each of these laminated blocks, so that $f_n = d_n/\sum_{n'} d_{n'}$. But the order in which layers were added to the blocks is not important, as Backus's formulas (Backus 1962) for the constants show. For the overall behavior for the quasistatic (long wavelength) behavior of the system we are studying, Backus's results [also see Berryman (1998), Milton (2002), Berryman (2004a)] state that

$$\begin{aligned} c_{33} &= \left\langle \tfrac{1}{K+4\mu/3} \right\rangle^{-1}, & c_{13} &= c_{33} \left\langle \tfrac{K-2\mu/3}{K+4\mu/3} \right\rangle, \\ c_{44} &= \left\langle \tfrac{1}{\mu} \right\rangle^{-1}, & c_{66} &= \langle \mu \rangle, \\ c_{11} &= \tfrac{c_{13}^2}{c_{33}} + 4c_{66} - 4\left\langle \tfrac{\mu^2}{K+4\mu/3} \right\rangle, & c_{12} &= c_{11} - 2c_{66}. \end{aligned} \tag{16}$$

This bracket notation can be correctly viewed as a line integral along the symmetry axis x_3.

The bulk modulus for each laminated grain is that given by the compressional Reuss average K_R of the corresponding compliance matrix s_{ij} [the inverse of the usual stiffness matrix c_{ij}, whose nonzero components are shown in (15)]. The result is $e = e_{11} + e_{22} + e_{33} = \sigma/K_{\rm eff}$, where $1/K_{\rm eff} = 1/K_R = 2s_{11} + 2s_{12} + 4s_{13} + s_{33}$.

Even though $K_{\rm eff} = K_R$ is the same for every grain, since the grains themselves are not isotropic, the overall bulk modulus K^* of the random polycrystal does not necessarily have the same value as K_R for the individual grains (Hill 1952). Hashin-Shtrikman bounds on K^* for random polycrystals whose grains have hexagonal symmetry (Peselnick and Meister 1965; Watt and Peselnick 1980) show in fact that the K_R value lies outside the bounds in many situations (Berryman 2004a).

3.2 *Bounds for random polycrystals*

3.2.1 Voigt and Reuss bounds: hexagonal symmetry

For hexagonal symmetry, the nonzero stiffness constants are: c_{11}, c_{12}, $c_{13} = c_{23}$, c_{33}, $c_{44} = c_{55}$, and $c_{66} = (c_{11} - c_{12})/2$.

The Voigt (1928) average for bulk modulus of hexagonal systems is well-known to be

$$K_V = \left[2(c_{11} + c_{12}) + 4c_{13} + c_{33}\right]/9. \tag{17}$$

Similarly, for the overall shear modulus G^*, we have

$$G_V = \frac{1}{5}\left(G_{\rm eff}^v + 2c_{44} + 2c_{66}\right), \tag{18}$$

where the new term appearing here is essentially defined by (18) and given explicitly by

$$G_{\rm eff}^v = (c_{11} + c_{33} - 2c_{13} - c_{66})/3. \tag{19}$$

The quantity $G_{\rm eff}^v$ is the energy per unit volume in a grain when a pure uniaxial shear *strain* of unit magnitude [*i.e.*, $(e_{11}, e_{22}, e_{33}) = (1, 1, -2)/\sqrt{6}$], whose main compressive strain is applied to the grain along its axis of symmetry (Berryman 2004a,b).

The Reuss (1929) average K_R for bulk modulus can also be written in terms of stiffness coefficients as

$$\frac{1}{K_R - c_{13}} = \frac{1}{c_{11} - c_{66} - c_{13}} + \frac{1}{c_{33} - c_{13}}. \tag{20}$$

The Reuss average for shear is

$$G_R = \left[\frac{1}{5}\left(\frac{1}{G_{\rm eff}^r} + \frac{2}{c_{44}} + \frac{2}{c_{66}}\right)\right]^{-1}, \tag{21}$$

that defines $G_{\rm eff}^r$ – *i.e.*, the energy per unit volume in a grain when a pure uniaxial shear *stress* of unit magnitude [*i.e.*, $(\sigma_{11}, \sigma_{22}, \sigma_{33}) = (1, 1, -2)/\sqrt{6}$], whose main compressive pressure is applied to a grain along its axis of symmetry.

For each grain having hexagonal symmetry, two product formulas hold (Berryman 2004b): $3K_R G_{\rm eff}^v = 3K_V G_{\rm eff}^r = \omega_+\omega_-/2 = c_{33}(c_{11} - c_{66}) - c_{13}^2$. The symbols $\omega_\pm$ stand for the quasi-compressional and quasi-uniaxial-shear eigenvalues for the crystalline grains. Thus, it follows that

$$G_{\rm eff}^r = K_R G_{\rm eff}^v / K_V \tag{22}$$

is a general formula, true for hexagonal symmetry.

3.2.2 Hashin-Shtrikman bounds

It has been shown elsewhere (Berryman 2004a,b) that the Peselnick-Meister-Watt bounds for bulk modulus of a random polycrystal composed of hexagonal (or transversely isotropic) grains are given by

$$K_{PM}^{\pm} = \frac{K_V(G_{\rm eff}^r + \zeta_\pm)}{(G_{\rm eff}^v + \zeta_\pm)} = \frac{K_R G_{\rm eff}^v + K_V \zeta_\pm}{G_{\rm eff}^v + \zeta_\pm}, \tag{23}$$

where $G_{\rm eff}^v$ ($G_{\rm eff}^v$) is the uniaxial shear energy per unit volume for a unit applied shear strain (stress). The second equality follows directly from the product formula (22). Parameters $\zeta_\pm$ are defined by

$$\zeta_\pm = \frac{G_\pm}{6}\left(\frac{9K_\pm + 8G_\pm}{K_\pm + 2G_\pm}\right). \tag{24}$$

In (24), values of $G_\pm$ (shear moduli of isotropic comparison materials) are given by inequalities

$$0 \le G_- \le \min(c_{44}, G_{\rm eff}^r, c_{66}), \tag{25}$$

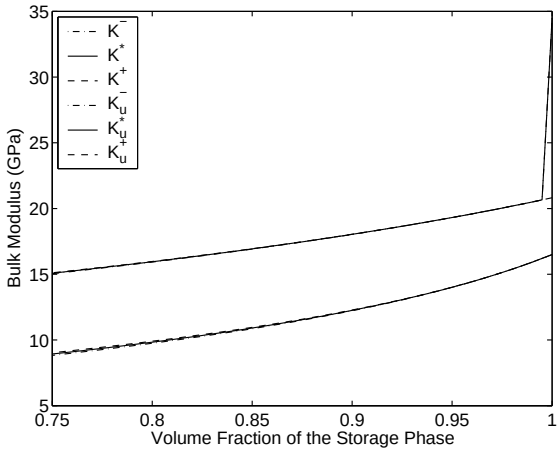

Figure 1: Bulk modulus bounds and self-consistent estimates for the random polycrystal of porous laminates model of a Navajo sandstone reservoir.

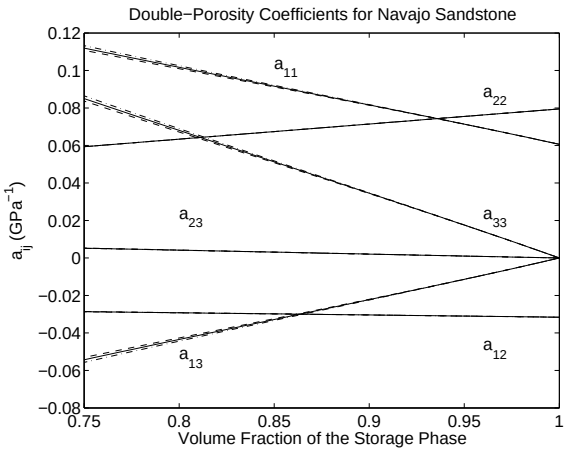

Figure 2: Values of double-porosity coefficients a_{ij} for a system similar to Navajo sandstone. Values used for the input parameters are listed in Table 1. For each coefficient, three curves are shown, depending on which estimate of the overall bulk modulus is used: lower bound (dot-dash line), self-consistent (solid line), or upper bound (dashed line).

and

$$\max(c_{44}, G^v_{\text{eff}}, c_{66}) \le G_+ \le \infty. \tag{26}$$

The values of $K_\pm$ (bulk moduli of isotropic comparison materials) are then given by algorithmic equalities

$$K_\pm = \frac{K_V(G^r_{\text{eff}} - G_\pm)}{(G^v_{\text{eff}} - G_\pm)}, \tag{27}$$

derived by Peselnick and Meister (1965) and Watt and Peselnick (1980). Also see Berryman (2004a).

Bounds on the shear moduli are then given by

$$\frac{1}{\mu^{\pm}_{\text{hex}}+\zeta_\pm} = \frac{1}{5}\left[\frac{1+\gamma_\pm(K_V-K_\pm)}{G^v_{\text{eff}}+\zeta_\pm+\delta_\pm(K_V-K_\pm)} + \frac{2}{c_{44}+\zeta_\pm} + \frac{2}{c_{66}+\zeta_\pm}\right], \tag{28}$$

where $\gamma_\pm$ and $\delta_\pm$ are given by

$$\gamma_\pm = \frac{1}{K_\pm + 4G_\pm/3}, \ \delta_\pm = \frac{5G_\pm/2}{K_\pm + 2G_\pm}. \tag{29}$$

K_V is the Voigt average of the bulk modulus as defined previously.

TABLE 1. Input Parameters for Navajo Sandstone Model of Double-Porosity System.

K_s	$K_s^{(1)}$	$K^{(1)}$	$K_s^{(2)}$	$K^{(2)}$
(GPa)	(GPa)	(GPa)	(GPa)	(GPa)
34.0	34.5	16.5	34.5	1.65
	$\nu^{(1)}$	$\phi^{(1)}$	$\nu^{(2)}$	$\phi^{(2)}$
	0.15	0.118	0.017	0.354

Note: Poisson's ratio ν and porosity ϕ are dimensionless.

4 EXAMPLE

Navajo sandstone is one possible host rock for which the required elastic constants have been measured (Coyner 1984). TABLE 1 displays the values needed in the double-porosity theory presented here.

The drained bulk moduli of the storage and fracture phases are used in the effective medium theory of Section 3 to determine the overall drained and undrained bulk moduli of the random polycrystal of laminates system. Results for the self-consistent estimates (Berryman 2004a) and the upper and lower bounds are all displayed in Figure 1. Observed dispersion is small over the range of volume fractions considered. Then these drained K values are used in the formulas of Section 2 to determine both estimates and bounds on the double-porosity coefficients. These results are displayed in Figure 2. The results for a_{11} essentially repeat results shown in Figure 1, but for the inverse of K^*.

The coefficients a_{12}, a_{22}, and a_{23} show little dispersion. This is natural for a_{12} and a_{22} because the storage material contains no fractures, and therefore is not sensitive to those mechanical effects on the overall reservoir. The behavior of a_{23} also shows little dispersion as this value is always quite close to zero. The two coefficients that show a significant level of dispersion are a_{13} and a_{33}, where the third stress is the pore pressure $p_f^{(2)}$ of the fracture or joint phase. We generally expect that the joint phase is most tightly coupled to, and therefore most sensitive to, the fluctuations in overall drained bulk modulus K^*. So all these results are qualitatively consistent with our intuition.

5 CONCLUSIONS

The methods presented have been successfully applied to determine geomechanical parameters for one reservoir model assuming Navajo sandstone is the host rock. Although the details differ, the basic ideas used above for elastic and poroelastic constants can also be used to obtain bounds and estimates of electrical formation factor and fluid permeability for the same random polycrystal of porous laminates model. Space constraints force us to pursue these issues further elsewhere.

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